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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
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Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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THE BEHAVIOR OF SHELLS MADE OF COMPOSITES DURING THE DYNAMIC APPLICATION OF AXIAL COMPRESSION

A. S. Vol'mir (*Moscow*) and L. N. Smetanina (*Voronezh*)

In nonlinear formulation we examine the problem of the dynamic stability of cylindrical shells made of composites, e.g., glass-reinforced plastic, under dynamic axial compression. We assume as given the law of mutual displacement of the ends of the shell. Using the Bubnov-Galerkin method, the equations of the nonlinear theory of shells in partial derivatives are reduced to ordinary differential equations with aperiodic coefficients. The equations obtained were integrated numerically using a BESM-2M computer for various loading rates and shell parameters relative to certain brands of glass-reinforced plastics. Similar problems for isotropic metal shells have been examined previously [1, 2]. The case of dynamic application of external pressure to an orthotropic shell was examined in [3].

Let us examine the behavior of a closed, round, cylindrical, reinforced-plastic shell subjected to dynamic axial compression (Fig. 1). We will solve this problem in the geometrically nonlinear formulation. Let us assume that the shell is attached by its ends with frames whose points can obtain certain radial displacements while the frames themselves remain curved. In view of the fact that the scatter of the experimental values of the critical loads is most influenced by the initial irregularities in the shape of the shell, let us study the behavior of shells that have initial camber. Let us use for the shell made of a composite a model of orthotropic design. We will consider that the principle directions of rigidity coincide with the generatrix of the cylinder and the cross-sectional arc. The elastic properties of orthotropic shells are characterized by moduli E_1 and E_2 in directions x and y, by shear modulus G, and by Poisson coefficients μ_1 and μ_2, which correspond to transverse strain along lines y and x. Let us mention the familiar relationship $E_1\mu_2 = E_2\mu_1$.

Let us select the coordinate system such that its origin coincides with a point belonging to the median surface. Coordinate x will be read along the generatrix, y – along the arc, and z – along the normal to the median surface; let us consider coordinate z as positive along the direction to the center of curvature. Let us designate the displacements of some point of the median surface in these directions as u, v, w. Let us write the equation of motion of an element of the shell, when the deflections are comparable with the thickness, in the presence of initial imperfections (when $q = 0$) [2]:

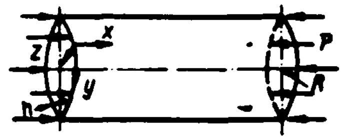

Fig. 1.

$$\frac{D_1}{h}\frac{\partial^4 (w-w_0)}{\partial x^4}+\frac{2D_3}{h}\frac{\partial^4 (w-w_0)}{\partial x^2\partial y^2}+\frac{D_2}{h}\frac{\partial^4 (w-w_0)}{\partial y^4}=$$
$$=\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 \Phi}{\partial x\partial y}+\frac{1}{R}\frac{\partial^2 \Phi}{\partial x^2}+\frac{q}{h}-\frac{\gamma}{g}\frac{\partial^2 w}{\partial t^2}. \qquad (1)$$

Here w and w_0 are the total and initial deflections; γ is the specific weight of the shell material; flexural rigidities in the axial and circular directions $D_{1,2} = E_{1,2}h^3/12(1 - \mu_1\mu_2)$; the derived rigidity $D_3 = D_1\mu_2 + 2D_G$; the torsional rigidity $D_G = Gh^3/12$; Φ – the stress function – is introduced by the formulas

$$\sigma_x=\frac{\partial^2\Phi}{\partial y^2},\quad \sigma_y=\frac{\partial^2\Phi}{\partial x^2};\tau=-\frac{\partial^2\Phi}{\partial x\partial y}.$$

To determine the stress function Φ let us use the equation of strain compatibility:

$$\delta_2\frac{\partial^4\Phi}{\partial x^4}+2\delta_3\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\delta_1\frac{\partial^4\Phi}{\partial y^4}=-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2+$$
$$+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2-\frac{1}{R}\frac{\partial^2 (w-w_0)}{\partial x^2}, \qquad (2)$$

where

$$\delta_{1,2}=1/E_{1,2};\quad 2\delta_3=1/G-2\mu_1/E_1.$$

We will not take into account the inertial forces corresponding to displacements of u and v in the median surface. Thus, in this solution we refrain from studying the propagation of elastic waves.

Let us use the problem solution plan as in [1, 3]. The total deflection is approximated as follows:

$$w=f(\sin\alpha x\sin\beta y+\psi\sin^2\alpha x\sin^2\beta y), \qquad (3)$$

where $\alpha = m\pi/L$; $\beta = n/R$; m is the number of half-waves along the generatrix; n is the number of waves around the perimeter of the shell. The expression

for initial deflection is obtained from (3) by substituting f_0 for f.

Let us consider that the shape of the initial wave formation, characterized by the parameter ψ, is "in resonance" with wave formation of the shell during strain, and that the only preassigned parameter is the value of f_0. Such an assumption intensifies somewhat the influence of the initial inaccuracies.

The boundary conditions for deflection w will be:

$$w = 0;\ \partial^2 w/\partial x^2 = 0 \text{ when } x = 0,\ L.$$

Substituting function (3) into the right side of strain compatibility equation (2), we get the integral of this equation in the form

$$\begin{aligned}\Phi = {} & \left[C_1 (f^2 - f_0^2)(1+\psi^2) + C_1' f_1 \psi\right] \cos 2\alpha x + C_2 (f^2 - f_0^2)(1+\psi^2) \cos 2\beta y + \\ & + C_3 (f^2 - f_0^2)\psi^2 \cos 4\alpha x + C_4 (f^2 - f_0^2) \psi^2 \cos 4\beta y + \left[C_5 f_1 + C_5' (f^2 - f_0^2)\psi\right] \times \\ & \times \sin \alpha x \sin \beta y + \left[C_6 (f^2 - f_0^2)\psi^2 + C_6' f_1 \psi\right] \cos 2\alpha x \cos 2\beta y + C_7 (f^2 - f_0^2) \times \\ & \times \psi^2 \cos 4\alpha x \cos 2\beta y + C_8 (f^2 - f_0^2)\psi^2 \cos 2\alpha x \cos 4\beta y + C_9 (f^2 - f_0^2)\psi \sin 3\alpha x \sin \beta y + \\ & + C_{10} (f^2 - f_0^2)\psi \sin \alpha x \sin 3\beta y - p y^2/2. \end{aligned} \tag{4}$$

In the expression for Φ we introduce the term $py^2/2$, which corresponds to basic compressive stress p.

Parameters $C_1 \ldots C_{10}$ depend on the values of α, β which characterize wave formation of the shell.

Let us solve Eq. (1) using the Bubnov-Galerkin method. As a result, we get equations which connect the deflection parameters with the time-changing load:

$$\begin{aligned}\hat{p}\zeta = {} & A_1(\zeta - \zeta_0) + A_2 \zeta(\zeta^2 - \zeta_0^2) + A_3 \psi^2 \zeta (\zeta^2 - \zeta_0^2) + A_4 \psi \zeta (\zeta - \zeta_0) + \\ & + A_5 \psi (\zeta^2 - \zeta_0^2) + \gamma g^{-1} (\partial^2 \zeta / \partial t^2) R^2 \left(\xi^2 \eta \sqrt{E_1 E_2}\right)^{-1}; \end{aligned} \tag{5}$$

$$\begin{aligned}\hat{p}\zeta\psi + {} & B_1' \zeta (\zeta - \zeta_0) + B_2' (\zeta^2 - \zeta_0^2) - B_3' \zeta (\zeta^2 - \zeta_0^2)\psi^2 + B_4' (\zeta^2 - \zeta_0^2)\psi^2 + \\ & + B_5' (\zeta - \zeta_0)\zeta\psi^2 - B_6' \zeta (\zeta^2 - \zeta_0^2)\psi - B_7' (\zeta - \zeta_0)\psi = 0. \end{aligned} \tag{5a}$$

In these equations the following dimensionless parameters and designations are introduced:

$$\zeta_0 = f_0/h;\ \zeta = f/h;\ \hat{p} = p R/\sqrt{E_1 E_2}\, h;\ \xi = m\pi R/nL;\ \eta = n^2 h/R.$$

The values of $A_1 \ldots A_5$ and $B_1' \ldots B_7'$ correspond to parameters ξ, Δ, which

characterize the form of wave formation of a shell and the mechanical properties of the material. Solving the problem in the first approximation, in Eq. (5) we drop the inertia term and use for parameter ψ the expression that results from Eqs. (5) and (5a). Relative shortening of the shell along the generatrix is defined by the formula

$$e=-\frac{1}{L}\int_0^L \frac{\partial u}{\partial x}\,dx=-\frac{1}{L}\int_0^L\left[E_1\left(\frac{\partial^2\Phi}{\partial y^2}-\mu_1\frac{\partial^2\Phi}{\partial x^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]dx. \quad (6)$$

Substituting into (6) the expressions for w, w_0 and Φ, and converting to dimensionless parameters, we get

$$\hat{e}=\hat{p}\Delta+\eta(\zeta^2-\zeta_0^2)\xi^2/8+3\eta(\zeta^2-\zeta_0^2)\xi^2\psi^2/32,\quad \Delta=\sqrt{E_2/E_1}. \quad (7)$$

Henceforth we will consider that the value of e changes in proportion to time $e = c't/L$, where c' is the rate of convergence of the ends, in m/s. Let us turn to dimensionless parameters

$$\hat{e}=c'tR/Lh;\quad \hat{t}=c'tR/LhA_1=\hat{e}/A_1. \quad (8)$$

Dividing Exp. (7) by A_1 and substituting the obtained dependence, and also substituting (8) into (7), we get

$$\frac{d^2\zeta}{d\hat{t}^2}-S_1\left\{\left[\hat{t}-\left(\frac{\eta\xi^2}{8A_1}+\frac{A_2\Delta}{A_1}\right)(\zeta^2-\zeta_0^2)\right]\frac{\zeta}{\Delta}-\frac{3\eta\zeta(\zeta^2-\zeta_0^2)\xi^2\psi^2}{32A_1\Delta}-\right.$$
$$\left.-(\zeta-\zeta_0)-\frac{A_3\zeta(\zeta^2-\zeta_0^2)\psi^2}{A_1}-\frac{A_4\zeta(\zeta-\zeta_0)\psi}{A_1}-\frac{A_5(\zeta^2-\zeta_0^2)\psi}{A_1}\right\}=0, \quad (9)$$

where

$$S_1=A_1^3(c')^{-2}(h/R)^2(L/R)^2\eta\sqrt{E_1E_2}\,\xi^2g/\gamma.$$

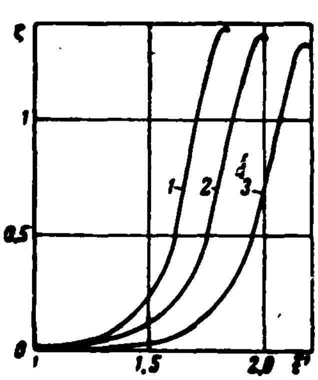

Fig. 2.

Equation (9) was integrated by the Runge-Kutta method using a BESM-2M computer, with the following starting data: $\zeta = \zeta_0$; $d\zeta/d\hat{t} = 0$ with $\hat{t} = 0$. Calculations were performed for shells with initial camber $\zeta_0 = 0.001$ relative to various brands of reinforced plastics. During the calculation we varied the shell parameters; the value of ξ was taken as 2, 3.5, and 5; the convergence rate of the ends was taken constant and equal to $c' = 3.5$ m/s. All curves were constructed so that parameter $\hat{t}' = \hat{t}A_1/0.605 = c'RT/0.605\,Lh$ was taken as the base.

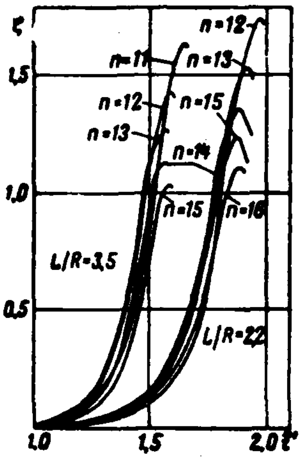

Fig. 3.

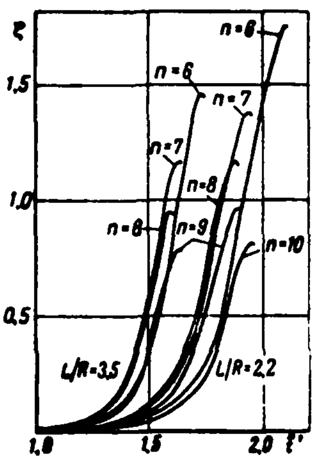

Fig. 4.

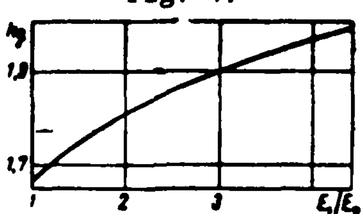

Fig. 5.

Figure 2 gives functions $L(\hat{t}')$ for shells with parameters $L/R = 2.2$; $h/r = 1/250$ for various ratios E_1/E_2 in the case $\xi = 2$. As is known, $\hat{t}' = 1$ corresponds to the convergence of the ends $\hat{e}_u$ at which the upper critical pressure is reached. From the figure it is evident that the rapid increase in deflections begins earliest at curve 1, for which $E_1 = E_2 = 1$, $n = 14$, then curve 2 with $E_1 = E_2 = 2$, $n = 15$, and curve 3 with $E_1 = E_2 = 5$, $n = 14$. Figures 3 and 4 give analogous curves for shells with parameters $L/R = 3.5$ and 2.2; $h/R = 1/250$ when $E_1 = E_2 = 1$. In Fig. 3 (when $\xi = 1$) in the case $L/R = 3.5$ the curve for which $\xi = 1.3$ deviates earliest from the abscissa; for the case $L/R = 2.2$ the curve to the left corresponds to $n = 14$. In Fig. 4 (when $n = 1$) the curve to the left corresponds to $n = 7$ in the case $L/R = 3.5$ and when $n = 8$ in the case $L/R = 2.2$. Figure 5 shows the dynamic coefficient k_g vs. the ratio $E_1 = E_2$ for shells with parameters $L/R = 2.2$; $h/R = 1/250$, $n = 2$. As can be seen from the figure, the lift of a shell increases with increasing ratio $E_1 = E_2$. With an increase in the parameter n the number of depressions along the peripheral arc decreases. With increasing ratio R/h the dynamic coefficient increases. Thus, the dynamic effect during the axial loading of a cylindrical shell made of a composite is expressed in the appearance of higher forms of stability loss and an increase in the critical load parameter. With an increase in the degree of anisotropy, corresponding to an increase in shell rigidity in the longitudinal direction, the dynamic coefficient increases.

REFERENCES

1. В. Л. Агамиров, А. С. Вольмир. Поведение цилиндрических оболочек при динамическом приложении всестороннего давления и осевого сжатия. «Известия АН СССР. Механика и машиностроение», 1960, № 3.

2. О. И. Теребушко. Устойчивость цилиндрической оболочки при кручении, внешнем давлении и сжатии. В сб.: «Расчет пространственных конструкций», вып. 4, Госстройиздат, 1958.

3. А. С. Вольмир, Л. Н. Сметанина. Исследование динамической устойчивости стеклопластиковых оболочек. «Механика полимеров», 1968, № 1, 109—116.

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